Instructions - Part 1

This qualifying exam is scored based on the answers to 25 problems. Each problem is worth 4 points, and partial credit will be awarded for work done on a problem.

The problems are divided into six areas of physics, and three areas will be covered in each two hour period. In each area there is a choice of problems to solve. If additional problems are attempted the best results will be selected for the score on the test.

Part 1 (2 hours)

Special Relativity (4 points): solve 1 of 2 problems.

Classical Mechanics (24 points): solve 6 of 8 problems.

Electromagnetism and Electronics (24 points): solve 6 of 8 problems.

You may use a calculator to find specific values required for some problems.

Useful constants for part 1:

$g = 9.8\ \mathrm{m/s^2}$

$G = 6.67 \times 10^{-11}\ \mathrm{N\ m^2 / kg^2}$

$R = 8.134$ J/mol-K

$c = 3.00 \times 10^{8}$ m/s

$e = 1.60 \times 10^{-19}$ C

$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2 / N.m^2}$

$\mu_0 = 4\pi \times 10^{-7}$ T m / A

$m_\mathrm{e} = 0.511\ \mathrm{MeV}/c^2$, $m_\mathrm{p} = 938\ \mathrm{MeV}/c^2$

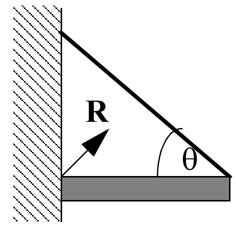
Special Relativity: complete 1 problem

1. A charged pion travels 30.0 m in the lab and decays in 1.00×10^{-8} s in its own rest frame. Find the speed of the pion in m/s.

2. A lump of clay whose rest mass is 4 kg is traveling at three-fifths the speed of light when it collides head-on with an identical lump going the opposite direction at the same speed. If the two lumps stick together and no energy is radiated away, find the mass of the composite lump in kg.

Classical Mechanics: complete 6 problems

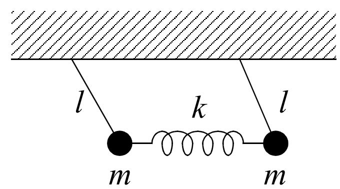
1. A boom is suspended by a cable as shown at right. Given that the angle $\theta = 45^{\circ}$ and the weight of the boom is $W = 1000$ N, find the magnitude and angle of the reaction force **R**.



2. Consider an object dropped near the earth's surface described in a coordinate system fixed with respect to the earth and up taken as positive. Air resistance is proportional to the velocity. In that coordinate system the equation of motion is given by $m\dfrac{dv}{dt} = -mg - bv$. Write the equation of motion for the object in a coordinate frame moving at a vertical velocity v_0 with respect to the fixed frame.

3. A bullet of mass 5.00 g has a velocity of 2.00×10^4 cm/s. It strikes a wooden block of 1.00 kg hanging as a pendulum on a rope attached to the ceiling. If the bullet becomes embedded in the block, find the maximum height (in cm) that the block rises after impact.

4. Consider two identical pendulums of mass m and length l connected by a massless spring of spring constant k as shown at right. Make an approximation of the motion along the horizontal axis and find the higher angular frequency of oscillation.



5. A stunt plane flies a loop-the-loop circle at uniform speed v. The pilot experiences an apparent weight at the bottom that is twice his apparent weight at the top of the circular path. What is the radius of the path?

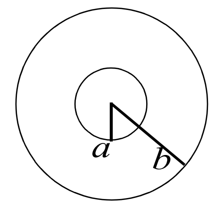
6. A cylinder with mass M and radius R has a radially dependent density. The cylinder rolls without slipping down a uniform inclined plane starting from rest at height h. At the bottom the translational velocity of the cylinder is $(8gh/7)^{1/2}$. Find the moment of inertia of the cylinder.

7. A balloon of negligible mass is filled with helium (density 0.18 kg/m^3). The balloon is holding up a mass of 300 kg in air (density 1.29 kg/m^3). Find the volume of the balloon.

8. A particle of mass m moves in a plane under the influence of a force $F = -kr$ directed toward the origin. Find the Lagrangian of the system using polar coordinates r, θ.

Electromagnetism and Electronics: complete 6 problems

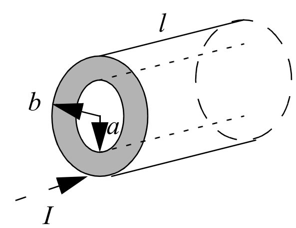
1. A circular annulus of inner radius a and outer radius b is centered at the origin in the xy plane. The annulus (between a and b) is filled with positive charge of density σ (charge/area). Find an expression for the electric field along the x-axis as a function of x.



2. Find the work required in keV to move 6.0 nC of charge against a potential of 1.1 μV.

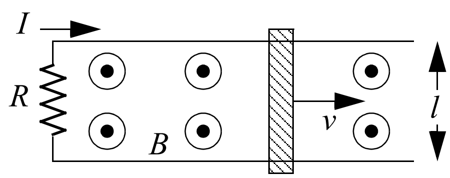
3. A capacitor is constructed from two metal plates of area A separated by a vertical distance d, and has a capacitance C_0 with no dielectric. Suppose that the upper half of the space between the plates is filled by a dielectric κ_1 and the lower half is filled by a dielectric κ_2. Find the capacitance C.

4. A resistor is made from a hollow cylinder of length l, inner radius a, and outer radius b. The region $a < r < b$ is filled with material of resistivity ρ. Find the resistance R of the cylinder.



5. A TTL logic gate is designed to be a NAND gate using positive logic. Find the truth table and determine the type of logical function assuming negative logic.

6. The circuit in the following figure has a resistor with $R = 2\ \Omega$ and is completed by a sliding conducting bar of length $l = 0.5$ m. The entire circuit is in a uniform magnetic field $B = 1$ T. Find the speed v of the conducting bar in m/s that maintains a current of $I = 0.5$ A.



7. A resistor R, capacitor C, and inductor L are in series with a voltage source supplying $V(t) = V_0 \cos\omega t$, where V_0 is a constant. Find the value of ω in terms of R, C and L, such that the current has the maximum steady state amplitude after transients have died out.

8. Write the classical Lagrangian for a particle of mass m, charge q and velocity $\boldsymbol{v}$ subject to an electric field $\boldsymbol{E}$ due to a potential ϕ and a magnetic field due to a vector potential $\boldsymbol{A}$ such that $\vec{B} = \nabla \times \vec{A}$.

Instructions - Part 2

This qualifying exam is scored based on the answers to 25 problems. Each problem is worth 4 points, and partial credit will be awarded for work done on a problem.

The problems are divided into six areas of physics, and three areas will be covered in each two hour period. In each area there is a choice of problems to solve. If additional problems are attempted the best results will be selected for the score on the test.

Part 2 (2 hours)

Quantum Mechanics (12 points): solve 3 of 4 problems.

Atomic, Nuclear, and Particle Physics (12 points): solve 3 of 4 problems.

Waves and Optics (12 points): solve 3 of 4 problems.

Thermodynamics and Statistical Physics (12 points): solve 3 of 4 problems.

You may use a calculator to find specific values required for some problems.

Useful constants for part 2:

$c = 3.00 \times 10^{8}$ m/s

$h = 6.63 \times 10^{-34}$ J s $= 1.24 \times 10^{-6}$ eV m/c; $hc = 1240$ eV nm

$m_e = 0.511$ MeV/c^2, $m_p = 938$ MeV/c^2

$N_A = 6.02 \times 10^{23}$ mol^{-1}

$k = 1.38 \times 10^{-23}$ J/K

$R = 8.314$ J / mol K

$\sigma = 5.67 \times 10^{-8}$ W / m^2 K^4

Quantum Mechanics: complete 3 problems

1. A photon and an electron each have a wavelength of 0.5 nm. Find the ratio of the momentum of the photon to the momentum of the electron.

2. The solution to the Schrodinger equation for the ground state of hydrogen is $\psi_0 = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ where a_0 is the Bohr radius and r is the distance from the origin. Find the most probable value of r.

3. Find the value of the commutator $[H, x]$ for the quantum mechanical Hamiltonian $H = p^2/2m + V(x)$.

4. A particle of energy $E < V_0$ is incident on a step potential of height V_0. Let $k = \sqrt{2mE}/\hbar$ and $k' = \sqrt{2m(V_0 - E)}/\hbar$. Find the transmission coefficient in terms of k and k'.

Atomic, Nuclear, and Particle Physics: complete 3 problems

1. Consider a hydrogen-like atom with a nuclear charge of Ze and a single electron of charge $-e$. The electron has principle quantum number n and orbital quantum number l. Find the degeneracy of the energy level in terms of n and/or l.

2. Use the Fermi gas model for electrons in a metal to find the Fermi wave vector $k_F = p_F/\hbar$ (in nm^{-1}) for electrons in a metal of density 0.971 g/cm^3 and molar mass 22.99 g/mol.

3. The binding energy of a heavy nucleus is about 7 MeV per nucleon, whereas the binding energy of a medium weight nucleus is about 8 MeV per nucleon. Use that information to estimate the total kinetic energy (in MeV) liberated when a heavy nucleus (such as U^{238}) undergoes symmetric fission.

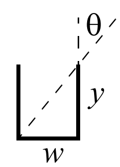
4. A photon strikes an electron of mass m that is initially at rest, creating an electron-positron pair. The photon is destroyed and the positron and two electrons move off at equal speeds along the initial direction of the photon. Find the energy of the photon.

Waves and Optics: complete 3 problems

1. The ionosphere may be viewed as a dielectric medium of refractive index $n = n(\omega_p)$, where ω_p is the plasma frequency. Calculate the phase velocity of a radio wave of frequency $\omega = \sqrt{2}\omega_p$ as a fraction of c.

2. At room temperature a pipe open at both ends resonates at a fundamental frequency f. On a particularly cold day the speed of sound is reduced by a small factor ε from v to $v(1\text{-}\varepsilon)$ from the speed at room temperature. The pipe is closed at one end. Find the fundamental frequency at which the closed pipe resonates on that day in terms of f and ε.

3. By looking at an empty glass of width w = 5.00 cm along the ray path θ shown at right, one sees the lower left hand corner. When the glass is filled with a clear liquid of refractive index n = 1.30, one sees the middle of bottom of the glass when viewed at the same angle θ. Find the height y of the glass.



4. Two slits are placed 2 mm apart and 300 cm from a screen. The slits are illuminated by light of 600 nm. Find the distance between bright lines of the interference pattern on the screen.

Thermodynamics and Statistical Physics: complete 3 problems

1. Two samples of an identical fluid are mixed. Sample 1 has a mass $5m$ at 60^{o}C and sample 2 has a mass m at -30^{o}C. Find the equilibrium temperature.

2. Consider black-body radiation in a cubical cavity of length L at a temperature T. Now assume that the length of each side of the cavity is doubled, and the temperature of the radiation and walls of the cavity is halved. Find the ratio of the total energy of the radiation in the cavity after the change compared to the initial state.

3. A three-dimensional harmonic oscillator is in thermal equilibrium with a temperature reservoir at temperature T. Find the total average energy of the oscillator.

4. A system of N independent particles with two spin states are in a magnetic field B where the energies of the two states are $E_+ = -\mu_0 B$ and $E_- = +\mu_0 B$. Assume the particle wavefunctions are not overlapping. Find the total average energy at temperature T.